Random partitions, tableaux and matrices : β-deformations and local limits

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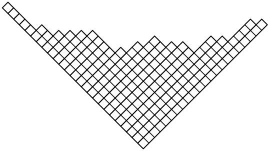
Workshop "Random matrices meet random permutations",
Lille, April 2022

Introduction: two different models with similar behaviour

Plancherel measure on partitions



For a partition λ, we take

$$\mathbb{P}(\lambda) = \frac{\dim(\lambda)^2}{n!}$$

GUE model of random matrices

$$\begin{pmatrix} a_{1,1} & a_{1,2} & \cdots \\ \overline{a_{1,2}} & \ddots & \vdots \\ \vdots & \cdots & a_{n,n} \end{pmatrix}$$

Hermitian matrix with independent complex Gaussian entries above the diagonal and real Gaussian entries on the diagonal.

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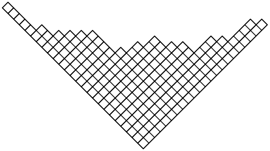

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Theorem (Borodin–Okounkov–Olshanski, Okounkov, Johansson, $\sim$'00)

Suitably renormalized, for all k, the first rows $(\lambda_1, \lambda_2, \dots, \lambda_k)$ of a random Plancherel Young diagram have the same fluctuations as the largest eigenvalues of a GUE matrix, i.e. they converge to the "Airy ensemble".

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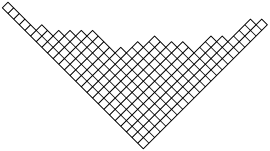

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Goal of the talk: discuss some other similarities and β-deformations.

Fixed dimension (after Śniady, '06)

Fix an integer $d \geq 1$ and consider a Plancherel random Young diagram **conditioned to have at most d rows**.

Permutation interpretation: we look at the RS shape of a uniform random permutation without decreasing subsequence of length $d+1$ (i.e. avoiding the pattern $d{+}1\ d\ \cdots\ 1$)

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Theorem (Śniady, '06)

Let $\lambda_n = (\lambda_{n,1}, \dots, \lambda_{n,d})$ *be a Plancherel random Young diagram* **conditioned to have at most** d **rows**. *Then*

$$\left(\sqrt{\frac{d}{n}}\, (\lambda_{n,i} - \frac{n}{d}) \right)_{1 \leq i \leq d}$$

converges in distribution to the eigenvalues of a **traceless** *GUE* $d \times d$ *random matrix.*

β-deformation (matrix side)

The eigenvalue of GUE random matrices have the following density w.r.t. Lebesgue measure on $\{x_1 \geq x_2 \geq \cdots \geq x_d\}$:

$$\frac{1}{C_d} e^{-(x_1^2+\cdots+x_d^2)} \prod_{i<j} (x_i - x_j)^2.$$

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We **define** GβE ensemble as having the following density w.r.t. Lebesgue measure on $\{x_1 \geq x_2 \geq \cdots \geq x_d\}$:

$$\frac{1}{C_d(\beta)} e^{-\frac{\beta}{2}(x_1^2+\cdots+x_d^2)} \prod_{i<j} (x_i - x_j)^\beta.$$

$\beta = 1, 4$: these are eigenvalues of natural models of matrices with real/quaternionic entries.

$\rightarrow$ huge literature on this model...

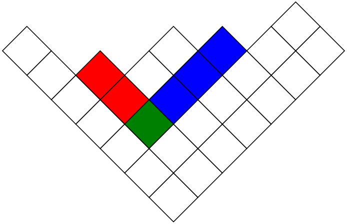
β-deformation (permutation side)

The usual Plancherel mesure is defined by

$$\mathbb{P}(\lambda) = \frac{\dim(\lambda)^2}{n!} = \frac{n!}{h_\lambda^2},$$

where

$$h_\lambda = \prod_{(i,j)\in\lambda} \big((\lambda_i - j) + (\lambda'_j - i) + 1\big)$$



β-deformation (permutation side)

The Jack-Plancherel mesure is defined by

$$\mathbb{P}(\lambda) = \frac{\alpha^n n!}{h_\lambda^{(\alpha)} h_\lambda^{\prime(\alpha)}},$$

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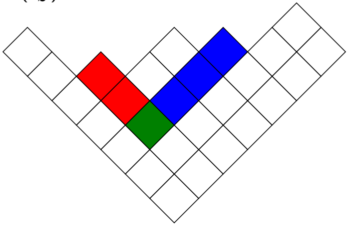

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Remark: $\mathbb{P}(\lambda) = [J_\lambda^{(\alpha)}]p_1^n$, where $J_\lambda^{(\alpha)}$ is the (integral) Jack polynomial indexed by λ.

A β version of Śniady's result (after Matsumoto, '08)

Theorem (Matsumoto, '08)

Let $\lambda_n^{(\alpha)} = (\lambda_{n,1}^{(\alpha)}, \dots, \lambda_{n,d}^{(\alpha)})$ be a Jack-Plancherel random Young diagram **conditioned to have at most d rows**. *Then*

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Permutation interpertation: for $\alpha = 2$, $\lambda_{n,1}^{(\alpha)}$ has the same distribution as the LIS of a uniform random fixed-point free involution conditionned to have no decreasing subsequence of length $> 2d$.

Transition

The Jack-Plancherel measure seems to be a nice analogue of $G\beta E$ models, at least in the fixed dimension setting.

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The Jack-Plancherel measure seems to be a nice analogue of $G\beta E$ models, at least in the fixed dimension setting.

$\rightarrow$ But what about the unconditioned version? We will see some results for fluctuations of linear statistics and edge fluctuations.

Fluctuations of linear statistics

Let M be a GUE random matrix, with eigenvalues $x_1 \geq x_2 \geq \cdots \geq x_n$. We let

$$\mu_M = \frac{1}{n} \sum_{i=1}^{n} \delta_{x_i}$$

be its empirical eigenvalue distribution. Then (Wigner, '58), for all k

$$\frac{1}{n} \mathrm{Tr}(M^k) = \int_{-2}^{2} x^k \mu_M(dx) \to_P \int_{-2}^{2} x^k \mu_{s-c}(dx),$$

where $\mu_{s-c}(dx) = \frac{1}{2\pi}\sqrt{4-x^2}dx$.

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Theorem (Johansson, '98)

Let T_k be a Chebyshev polynomial of degree k. Then, jointly for all k

$$\operatorname{Tr}(T_k(M)) - n \int_{-2}^{2} T_k(x) \mu_{s-c}(dx) \to_d \frac{\sqrt{k}}{2} \xi_k,$$

where ξ_k are independent Gaussian variables.

Fluctuations of linear statistics (reformulated)

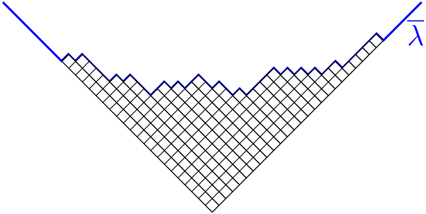
Another formulation of Johansson's theorem (Ivanov, Olshanski, '03):

$$\mu_M \equiv \mu_{s-c} + \frac{1}{n}\widetilde{\Delta}(x) + o\big(n^{-1}\big),$$

where $\equiv$ means that equality holds when integrated over any polynomial function of x and

$$\widetilde{\Delta}(2\cos\theta) = \frac{1}{2\pi}\sum_{k\geq 1} \frac{\sqrt{k}\xi_k \cos(k\theta)}{\sin(\theta)}.$$

Fluctuations of linear statistics – the partition case



Limit shape result for Plancherel Young diagrams:
(Kerov–Vershik/Logan–Shepp '77)

$$\sup_{x \in \mathbb{R}} |\overline{\lambda}_n(x) - \Omega(x)| \to_P 0.$$

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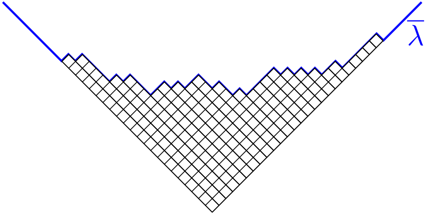


Theorem (Ivanov–Olshanski '03, based on unpublished notes by Kerov)

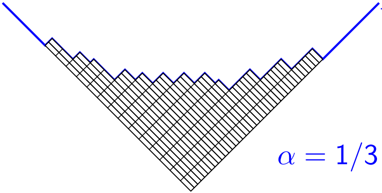
Let λ_n be a Plancherel random Young diagram of size n and $\overline{\lambda}_n$ its renormalized upper boundary . Then

$$\overline{\lambda}_n \equiv \Omega(x) + \frac{2}{\sqrt{n}}\Delta(x) + o\left(n^{-1/2}\right),$$

where

$$\Delta(2\cos\theta) = \frac{1}{2\pi}\sum_{k\geq 1}\frac{\xi_k \sin(k\theta)}{\sqrt{k}}.$$

Fluctuations of linear statistics – the β-partition case



Theorem (F. – Dołęga, '16)

Let λ_n be a Jack-Plancherel random Young diagram of size n and $\overline{\lambda}_n^\alpha$ its renormalized upper boundary (with rows scaled by $\sqrt{\frac{\alpha}{n}}$ and columns by $\frac{1}{\sqrt{\alpha n}}$). Then

$$\overline{\lambda}_n^\alpha \equiv \Omega(x) + \frac{2}{\sqrt{n}} \Delta^{(\alpha)}(x) + o\left(n^{-1/2}\right),$$

where

$$\Delta^{(\alpha)}(2\cos\theta) = \frac{1}{2\pi} \sum_{k \geq 1} \frac{\xi_k \sin(k\theta)}{\sqrt{k}} + \left(\sqrt{\alpha}^{-1} - \sqrt{\alpha}\right) \left(\frac{\theta}{2\pi} - \frac{1}{4}\right).$$

Comparison with β-ensemble

Our result is reminiscent of fluctuations of linear statistics of β-ensembles

Theorem (Dimitriu – Edelman, '06)

Let μ_β be the (random) empirical measure of a $G\beta E$ ensemble of n particles. Then

$$\mu_\beta \equiv \mu_{s-c} + \frac{\sqrt{\alpha}}{n}\widetilde{\Delta}(x) + \frac{\alpha-1}{n}\mu_H(dx) + o\left(n^{-1}\right),$$

where $\widetilde{\Delta}$ is the same generalized Gaussian process as before, and μ_H is the following deterministic signed measure:

$$\mu_H(dx) = \frac{1}{4}\delta_{-2} + \frac{1}{4}\delta_2 - \frac{1}{2\pi}\frac{dx}{\sqrt{4-x^2}}.$$

$\rightarrow$ For both partitions and matrices, the β-deformation adds a deterministic term. . .

Some proof ideas (only on the partition side, here $\alpha = 1$)

Character values: fix a permutation σ in S_k of cycle-type μ, and consider the function on Young diagrams

$$\mathrm{Ch}_\mu : \lambda \mapsto |\lambda| \dots (|\lambda| - k + 1) \frac{\chi^\lambda(\sigma)}{\dim(\lambda)}.$$

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Lemma

Fix $\mu \neq (1^r)$. If λ is taken at random with Plancherel measure of size n, then

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What about higher (joint) moments of Ch_μ? Can we compute $\mathrm{Ch}_\mu \cdot \mathrm{Ch}_\nu$?

Yes, using Plancherel's isomorphism, it is equivalent to multiply conjugacy classes in the symmetric group algebra!

Example: $\mathrm{Ch}_{(2)} \cdot \mathrm{Ch}_{(2)} = \mathrm{Ch}_{(2,2)} + 4\,\mathrm{Ch}_{(3)} + 2\,\mathrm{Ch}_{(1,1)}$

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Using this and the method of moments, one can prove

Theorem (Kerov, '93, Hora, '98)

$\frac{\mathrm{Ch}_{(k)}}{\sqrt{k} n^{k/2}}$ *converge jointly to independent Gaussian random variables.*

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What about the fluctuations of the rescaled diagram $\overline{\lambda}$? The connection goes through the following fact.

Proposition (Kerov–Olshanski, '94, stated informally)

The functions $(\mathrm{Ch}_{(k)})_{k \geq 2}$ *and* $\lambda \mapsto \int_{\mathbb{R}} x^k (\overline{\lambda}(x) - |x|) dx$ *generate the same algebra of functions on Young diagrams.*
(+ some formulas to go from one set of generators to the other one.)

Some proof ideas (only on the partition side, general α)

For general α, there is no representation theory behind the scene!
But we can define a nice deformation of Ch_μ:

$$\mathrm{Ch}^{(\alpha)}_\mu : \lambda \mapsto \alpha^{-\frac{|\mu|-\ell(\mu)}{2}}\ z_\mu\,[p_{\rho 1^{|\lambda|-k}}]J^{(\alpha)}_\lambda.$$

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- There exist coefficients $g_{\mu,\nu;\pi}^{(\alpha)}$, such that

$$\mathrm{Ch}_\mu^{(\alpha)} \cdot \mathrm{Ch}_\nu^{(\alpha)} = \sum_{\substack{\pi \text{ partition} \\ \text{of any size}}} g_{\mu,\nu;\pi}^{(\alpha)} \, \mathrm{Ch}_\pi^{(\alpha)},$$

  but we have no Plancherel isomorphism and thus no combinatorial description of these coefficients. :(

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- Using polynomial interpolation, we prove that the first four moments of $\frac{\mathrm{Ch}^{(\alpha)}_{(k)}}{n^{k/2}\sqrt{k}}$ coincide asymptotically with that of a Gaussian.
- We use Stein's method to prove the asymptotic normality (based on a work on Fulman for $\mathrm{Ch}^{(\alpha)}_{(2)}$, '04).

And edge fluctuations?

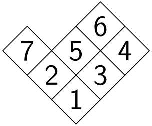
Theorem (Guionnet, Huang, '19)

Let λ^n be a Jack-Plancherel random Young diagram of size n and fix $k \geq 1$. Then the lengths of the k first row $(\lambda_1^n, \dots, \lambda_k^n)$ of λ^n has asymptotically the same fluctuations as the k first particles of a $G\beta E$ ensemble, where $\beta = 2/\alpha$.

Note: the combinatorially relevant cases $\alpha \in \{1/2, 2\}$ were proven earlier by Baik and Rains, '01.

Transition

We now add the time dimension on the partition side, and look at random tableaux.



$$\equiv \quad \emptyset \mapsto (1) \mapsto (1,1) \mapsto (2,1) \mapsto (3,1) \mapsto (3,2) \mapsto (3,3) \mapsto (3,3,1)$$

Model of random tableaux: fix the shape λ and take a uniform random tableau T of shape λ.

Some links with random matrices

Theorem (Marchal, '07)

Let T be a random Young diagram of square shape $n \times n$. Then, there exists a function $r(t)$ for $t \in (0,1)$, we have

$$\frac{r(t)\Big(T(1,\lfloor nt \rfloor) - \mathbb{E}T(1,\lfloor nt \rfloor)\Big)}{N^{4/3}} \to_d TW,$$

where TW is the GUE Tracy–Widom distribution.

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Theorem (Gorin–Rahman, '19)

Let T be a random Young diagram of staircase shape $(n-1, n-2, \dots, 1)$. Then, for $\alpha \in (-1;1)$, the entry $T\left(\lfloor \frac{(1+\alpha)n}{2} \rfloor, \lceil \frac{(1-\alpha)n}{2} \rceil\right)$ on the outer border as the same fluctuations as the smallest positive eigenvalue Λ_+ of a GOE matrix.

Local limits of Young tableaux (work in progress with J. Borga, C. Boutillier, P.-L. Méliot)

One can encode tableaux as particles (or beads) on vertical lines (threads), with an interlacing condition.

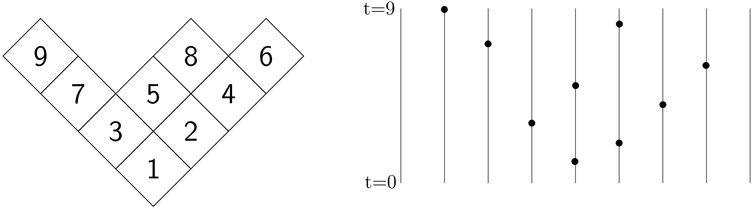

Figure: A Young tableau T and the associated set M_T.

We look at M_T around a point $(x_0\sqrt{n}, t_0\, n)$ in a window of size $O(1) \times O(\sqrt{n})$, i.e. we set

$$\widetilde{M_\lambda} = \Big\{(y, \varepsilon) \in \mathbb{Z} \times \mathbb{R} : \big(x_0\sqrt{N} + y, t_0 n + \varepsilon\sqrt{n}\big) \in M_{\lambda_N}\Big\}.$$

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What are we trying to prove?

When the size of λ goes to infinity with some limit shape, for (x_0, t_0) in the bulk, the renormalized process $\widetilde{M_\lambda}$ converges to Boutillier's bead model.

What is the limit object?

- Boutillier's bead models form a natural family of models of random interlacing bead configurations;
- They are local limits in the bulk of GUE corner processes (Adler, Nordenstam, Van Moerbeke, '14).

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- for Poissonized tableaux instead of standard ones;
- when λ is obtained by substituting each box by a $c \times c$ square in a base diagram λ_0 (making c tend to ∞);
- under a technical condition on (x_0, t_0).

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Main tool: The particle representation of Poissonized tableau is a determinantal point process! (Gorin–Rahman, '19)

β-deformation

Fact: the Jack-Plancherel measure is Markovian (i.e. one can sample a diagram of size n easily starting from one of size $n-1$).

$\rightarrow$ one can define β-random tableaux (Plancherel distributed, or of a given shape if one conditioned to the final shape).

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Future work?

- Do fluctutations of a Plancherel random β-tableau conditioned on having at most d rows converge to a β-Dyson Brownian motion?
- Is the local limit of β-tableaux in the bulk the β-bead model (Najnudel, Virag, '21)?

Thank you for
your attention!